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LOCAL CHARACTERISTICS OF HEAT-RADIATION SUPERCONDUCTOR DETECTORS BASED ON HIGH-TEMPERATURE SUPERCONDUCTOR FILMS

O. S. Esikov, A. I. Krot, I. G. Merinov,
E. A. Protasov, V. P. Sobolev,
and V. S. Kharitonov

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Relations are proposed for evaluating the local values of the thermal sensitivity, speed of response, and resolution of a heat radiation detector based on high-temperature superconductor films.

Superconductor sensitive elements (SE) find an ever wider application in creating heat radiation detectors [1]. Until recently, however, their application has been limited by the necessity of using liquid helium as a thermostabilizing coolant. The appearance of high-temperature superconductors (HTSC), whose superconducting transition temperature exceeds the boiling temperature of liquid nitrogen at normal pressure, has greatly extended the range of application of superconductive heat radiation detectors (SHRD). Study [2] analyzed the integral inertial-sensitive characteristics of SHRD manufactured based on HTSC films by various methods of their thermostabilization. It is shown that they are determined chiefly by the thermophysical properties of the substrate and by the value of the thermal coupling of the SE with the thermostat. The estimates obtained therein made it possible to draw a conclusion that the existing technologies permit the creation, based on HTSC films, of SEs operating at nitrogen temperatures, which could provide, with response times of about 1 sec, integral thermal response not worse that 10^3 K/W.

The detection of several heat radiation sources or the image reconstruction of an extended heat source raises the problems of the spatial and temporal resolution of heat fluxes incident on SE. The local thermal characteristics of the SEs of SHRDs also begin to play an important role. The aim of the present study is to analyze the basic local thermal characteristics (sensitivity, speed of response, and resolution) of the SEs of SHRDs based on HTSC films.

In the simplest of cases an SE is a flat multilayer structure whose main elements are an HTSC film, a substrate, a thermally-controlled coolant duct, and a thermal resistance providing for the thermal coupling of the substrate with the coolant duct (Fig. 1). The thickness of the HTSC film is usually two orders of magnitude smaller than the substrate thickness and is about 1 μm. Therefore, already 1 μsec after heat radiation starts affecting the SE, the substrate influence on a temperature mode of the HTSC film becomes dominant. To estimate local thermal characteristics of the SE, let us substitute for it a model of a semi-infinite medium with the same thermophysical parameters as the substrate and analyze its temperature field due to a narrowly-directed heat radiation flux incident on the surface (Fig. 2).

To describe the substrate temperature in the approximation adopted, we can use the known solution to a problem on the propagation of thermal energy into the half-space from the local heat source of power Q located on its surface in a circle of radius r_0 [3]:

$$\Delta T(r, z, t) = T(r, z, t) - T_0 = \frac{Q}{2\pi r_0 \lambda} \int_0^\infty J_0(\xi r) J_1(\xi r_0) \times \left(\exp(\xi z)\, \Phi\left(\frac{z}{2(at)^{0.5}} + \xi (at)^{0.5}\right) - \right.$$
$$\left. - \exp(-\xi z)\, \Phi\left(\frac{z}{2(at)^{0.5}} - \xi (at)^{0.5}\right) - 2\,\mathrm{sh}(\xi z)\right) \frac{d\xi}{\xi}. \quad (1)$$

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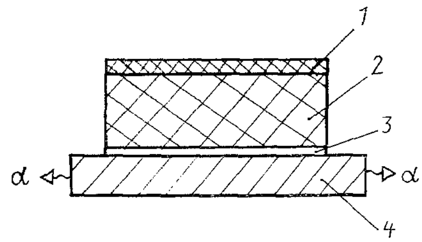

Fig. 1

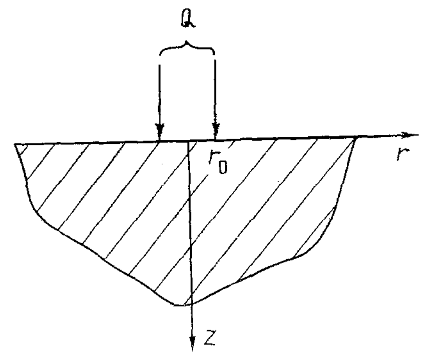

Fig. 2

Fig. 1. Diagram of SE: 1) HTSC film; 2) substrate; 3) thermal resistance; 4) coolant duct.

Fig. 2. Thermal model of SE.

The relation for a stationary distribution of the SE temperature in this approximation has the form

$$\Delta T_{st}(r,\ z) = \frac{Q}{\pi r_0 \lambda} \int\limits_0^\infty \exp(-\xi z) J_0(\xi r) J_1(\xi r_0) \frac{d\xi}{\xi}. \quad (2)$$

The maximum local thermal sensitivity S_{max} is achieved in the center of the heat emission spot at r = 0 and z = 0:

$$S_{max} = \frac{\Delta T_{st}(0,\ 0)}{Q} = \frac{1}{\pi r_0 \lambda}. \quad (3)$$

It follows from relation (3) that the static local thermal sensitivity for narrowly-directed heat fluxes does not depend on the SE geometric characteristics or on the conditions of its thermostabilization but is determined only by the thermal conductivity of the substrate and by the cross section of the incident radiation beam. It is evident that these deductions are valid only within the framework of the model of the semi-infinite medium, which can be used only in the case when the transverse size of the radiation beam incident on the SE is many times smaller than the substrate thickness. On the other hand, if the beam size becomes commensurate with or smaller than the thickness of the HTSC film, then relation (3) does not hold any longer either and the influence of the HTSC film should be taken into consideration. The static local thermal sensitivity of the SE with a substrate made of strontium titanate, calculated from equation (3) (λ = 4.5 W/mK at T = 90 K), for a beam 10 μm in diameter is approximately $1.4 \cdot 10^4$ K/W.

Following the definitions adopted in study [2], let us characterize the local speed of response β by the quantity which is the inverse of the time required for reaching half of the maximum excess temperature:

$$\beta = 1/\tau_{1/2}. \quad (4)$$

The temperature change in the center of the heat emission spot as a function of time is defined by the expression:

$$\Delta T(0,\ 0,\ t) = \frac{2Q(at)^{0.5}}{\pi^{1.5} r_0^2 \lambda} \left[1 - \exp\left(-\frac{r_0^2}{4at} \right) + \right.$$
$$\left. + \frac{r_0 \pi^{0.5}}{2(at)^{0.5}} \left[1 - \Phi\left(\frac{r_0}{2(at)^{0.5}} \right) \right] \right]. \quad (5)$$

TABLE 1. Thermophysical Parameters of the Film and Substrate [5-7]

Material	λ, W/(m·K)	c_p, J/kg·K	ρ, kg/m^3
Superconductor ($YBa_2Cu_3O_7$)	2,8	165	$5,25\cdot 10^3$
Substrate ($SrTiO_3$)	4,5	213	$6,45\cdot 10^3$

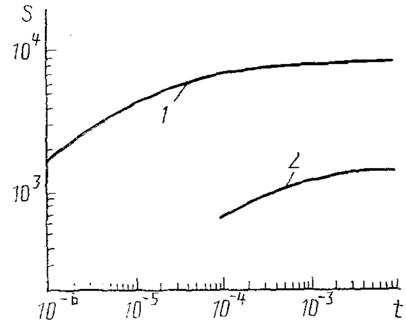

Fig. 3

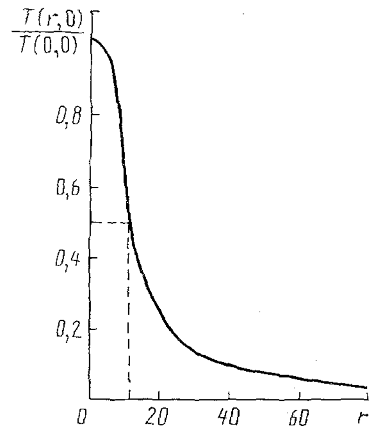

Fig. 4

Fig. 3. Time variation of thermal sensitivity S of SE under the influence of a local heat radiation source: 1) r_0 = 10 μm; 2) 50. S, K/W; t, sec.

Fig. 4. Stationary distribution of the relative temperature on the surface of an HTSC film subjected to a local heat radiation source of 10 μm radius. r, μm.

At $t = \tau_{1/2}$, the exponent and error integral arguments are smaller than 1. We can limit ourselves, therefore, by their approximate expressions, using the Taylor series expansion in terms of the small argument:

$$\exp(-x) \approx 1 - x + \frac{x^2}{2}, \tag{6}$$

$$\Phi(x) \approx \frac{2}{\pi^{0,5}}\left[x - \frac{x^3}{3}\right]. \tag{7}$$

Substituting (6) and (7) into (5) and taking into consideration that $\Delta T(0, 0, \tau_{1/2}) = \Delta T_{max}/2$, we obtain an equation defining $\tau_{1/2}$:

$$1 - \frac{r_0}{(\pi a\tau_{1/2})^{0,5}} + \frac{r_0^3}{24\pi^{0,5}(a\tau_{1/2})^{1,5}} = 0, \tag{8}$$

from which we find

$$\beta = \frac{1.65\pi a}{r_0^2}. \tag{9}$$

Thus, the local high-speed response depends only on the transverse size of the incoming beam and on the substrate thermal conductivity. The local speed of response of the SE with a substrate made of strontium titanate during its exposure to a beam 10 μm in diameter is approximately $7 \cdot 10^5$ sec^{-1}.

For defining the spatial resolution of the SE let us use the expression for the stationary temperature field on the sensing surface at z = 0:

$$\Delta T_{st}(r, 0) = \frac{Q}{\pi \lambda r_0} \int_0^\infty J_0(\xi r) J_1(\xi r_0) \frac{d\xi}{\xi} . \qquad (10)$$

A resolution of the temperature fields from two identical heat radiation beams located at a distance L from each other is possible in the case when at the point situated halfway between them the contribution to the temperature field from any of them does not exceed half of the maximum temperature in the center of the heat emission spot:

$$\int_0^\infty J_0(\xi L/2) J_1(\xi r_0) \frac{d\xi}{\xi} < 0{,}5. \qquad (11)$$

The integral on the left side of expression (11) can be transformed to complete elliptic integrals of the first and second kind [4], whose values decrease rapidly with a growing L, whereas condition (11) is fulfilled only for L greater than $2.3r_0$. Thus, the spatial resolution of the SE with respect to local heat radiation sources does not depend on the substrate material and is equal to 1.15 diameters of the heat emission spot.

The expressions obtained for the local thermal characteristics of the SE and the estimates of their values are valid practically for any point of the detecting surface since, due to the high spatial resolution the edge effects will be essential only within a narrow boundary area with a width of no more than $10r_0$.

More accurate estimates of the local thermal characteristics of a possible variant of the SE (see Fig. 1) were obtained with the help of a numerical model and of the program described in [2]. The behavior of a flat SE with an HTSC sensitive layer under the influence of a heat radiation beam 20 or 100 μm in diameter was studied in the process of modeling. The diameter of the substrate made of strontium titanate and the diameter of the HTSC film of $YBa_2Cu_3O_7$ were 20 mm. The substrate thickness and the film thickness were equal to 200 and to 1 μm, respectively. The total resistance of the thermal coupling of the substrate with the coolant duct varied from 1.6 to 16 K/W. The utilized values of the thermophysical parameters of the substrate and of the film are given in Table 1.

Figure 3 shows the time dependence of the ratio of the temperature change in the center of the heat emission spot to the power of the incident flux. This ratio can be considered as an instantaneous value of the local thermal response of the SE to the given radiation beam, whereas its maximum value gives the static local response. Differences of 10 to 15% from the analytical estimate based on formula (3) are connected, mainly, with the neglect of the influence of the HTSC film on the thermal response of the SE. The instantaneous response reaches half the static response value in 8 μsec after receiving a beam 20 μm in diameter and in 0.14 msec for a beam 100 μm in diameter. The speed of response value for a beam 100 μm in diameter, which equals $7 \cdot 10^3$ sec^{-1}, obtained from here, agrees well with the estimate given by relation (9). At the same time, the speed of response value for a beam 10 μm in diameter, which is equal to $1.25 \cdot 10^5$ sec^{-1} and was obtained from numerical calculations, differs from the analytical estimate by 26%. This difference is connected with the fact that the smaller the diameter of the heat radiation beam, the more significant is the influence of the HTSC film, which is not taken into consideration in formula (9).

Figure 4 shows the stationary distribution of the relative temperature (the ratio of the temperature at an arbitrary point to the temperature in the center of the heat emission spot) on the surface of the HTSC film. The SE resolution, calculated with its help, is close to 1 and also agrees well with preliminary estimates.

Our research has shown that the local inertial-sensitive thermal characteristics of the SEs of SHRDs can differ considerably from the mean integral ones and are defined chiefly by the heat transfer and the thermal conductivity of the substrate and by the cross section of the incident radiation beam.

NOTATION

T, substrate temperature; T_0, initial substrate temperature; r and z, radial and axial coordinates; t, time; Q, heat-radiation source power; λ, thermal conductivity; a, thermal diffusivity; r_0, radius of the heat radiation beam; J_0 and J_1, Bessel functions of the zero and first order; Φ, error integral; ΔT, excess temperature; T_{st}, stationary temperature; β, local speed of response; $\tau_{1/2}$, time required for reaching half of the maximum excess temperature; C_p, specific heat; ρ, density.

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NUMERICAL METHOD FOR ANALYZING A STOCHASTIC STATIONARY HEAT-CONDUCTION EQUATION WITH RANDOM COEFFICIENTS

A. G. Madera

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A numerical method is suggested for defining mathematical expectation fields and the variance of a stochastic temperature field which is described in the stationary case by a stochastic heat conduction equation and boundary conditions with random coefficients. Random coefficients of the stochastic mathematical model may obey arbitrary truncated distribution laws. An example of using the developed method is presented.

Introduction. Real temperature distributions in real objects are stochastic. This fact is caused by the randomness of the parameters and characteristics determining a temperature field. Such parameters and characteristics as powers of sources and sinks of heat, thermal conductivity coefficients of a body, coefficients of heat transfer from a body surface into a medium, environment temperature, gaps between contacting bodies, etc., may be random and have a significant statistical scatter. The stochasticity of these parameters and characteristics is a consequence of the random technological scatter and random fluctuations of the parameters characterizing heat transfer between the object and the medium.

In engineering practice the temperature mathematical-expectation and temperature variance fields are the most important probability characteristics of the stochastic temperature distribution in objects. Having available these probability characteristics, one can determine the fields of confidence intervals in the object. The real values of temperatures (which may occur in practice) will be arranged inside these intervals.

At present, there exist the following numerical methods for analyzing stochastic temperature fields in a body: perturbation theory methods [1]; the finite-element method for a differential equation with the coefficient of an unknown and the free term both being white Gaussian noise [2]; and the method of the stochastic Green's function [3, 4]. However, the perturbation theory methods are applicable only in the case when random fluctuations of the parameter are much smaller than

Institute of Cybernetics Problems of the Academy of Sciences of USSR, Moscow.

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